

UNIVERSITY OF NORTH CAROLINA AT CHARLOTTE
Department of Electrical and Computer Engineering

EXPERIMENT 9 – FOURIER SERIES

OBJECTIVES

After completing this experiment, the student will have

- Compose arbitrary periodic signals from a series of sine and cosine waves.
- Confirm the Fourier Series equation.
- Compute Fourier coefficients of a waveform.

MATERIALS/EQUIPMENT NEEDED

NI ELVIS II

EMONA SIGEx Signal & Systems add-on board

Assorted patch leads

Two BNC – 2mm leads

INTRODUCTION

A non-sinusoidal waveform could be expressed as a sum of sinewaves, over a suitable frequency range, either exactly or approximately. The system output can then be obtained by exploiting the additivity property of linear systems, i.e., first obtain the output corresponding to each sinusoidal component of the input signal, then take the sum of the outputs. In this experiment this idea is explored by means of a technique based on the generation of beat frequencies -- somewhat like when a musician uses a tuning fork. Will start by adding together many beat frequencies and view the resulting waveform. From there will look at the equations and use trigonometry to decompose waveforms into their constituent components.

Constructing Waveforms from Sines and Cosines

A simple sinusoid with zero phase can be represented by the equation

$$f(t) = \cos(\omega t) \tag{9.1}$$

And a cosine wave with a frequency of n times can be simply represented by the equation

$$f(t) = \cos(n\omega t) \tag{9.2}$$

Any wave which is an integer multiple of another frequency is known as a harmonic of the frequency. So equation (9.2) represents the harmonics of the signal in equation (9.1) for $n > 1$.

The general form of the equation for the summation of cosine harmonics is as follows:

$$f(t) = \sum_{n=1}^N a_n \cos(n\omega t) \tag{9.3}$$

where a_n are the amplitude values of each cosine wave.

The general form of the equation for the summation of sine wave harmonics is as follows:

$$f(t) = \sum_{n=1}^N b_n \sin(n\omega t) \quad (9.4)$$

where b_n are the amplitude values of each sine wave.

The general form for the summation of both sine and cosine harmonics is:

$$f(t) = \sum_{n=1}^N a_n \cos(n\omega t) + \sum_{n=1}^N b_n \sin(n\omega t) \quad (9.5)$$

Even functions are symmetrical about the Y axis. Odd functions are not symmetrical about the Y axis, but appear to be inverted about the X axis on the negative side of the Y axis. Sine and cosine waves, as well as their sums of sine and cosines, are always symmetrical about the X axis. Being symmetrical means that they cannot represent a DC offset. In order to have a DC offset we must add a constant to the equation.

If a_0 is added to represent that constant and the general equation for our arbitrary wave form becomes

$$f(t) = a_0 + \sum_{n=1}^N a_n \cos(n\omega t) + \sum_{n=1}^N b_n \sin(n\omega t) \quad (9.6)$$

where N is max number of harmonics present. In our experiment will $N=10$.

This equation describes any arbitrary wave form with the provision that it is periodic. Any periodic waveform, no matter how complicated, can be represented by the summation of many simple sine and cosine waveforms. This equation is known as the Fourier series equation.

When N equals one, this frequency known as the fundamental frequency, is also called the resolution frequency. Being the smallest frequency in this series it defines the minimum separation between components in a particular waveform. In the Fourier series equation above waves were grouped in terms of whether they are sine or cosine waveforms. This equation can be modified and grouped individual components by their harmonic number. This is easy enough and is as follows

$$f(t) = a_0 + \sum_{n=1}^N a_n \cos(n\omega t) + b_n \sin(n\omega t) \quad (9.7)$$

Grouping in this way allows us to think of each frequency as having both a sine and cosine component. Let us have a quick revision of what adding a sine and a cosine of the same frequency together may result in.

For each harmonic, $n\omega t$, a signal may have a sine and cosine component, implemented by their own respective amplitude, a_n and b_n . It is helpful to think about these component pairs as a single entity at a particular frequency. The only difference between them being their respective amplitude and the respective orientation or phasing. We know that a sine wave is 90° out of phase with a cosine wave, a quality which makes those two components orthogonal to each other.

By orthogonal we mean that they operate independently of each other. And so we need a notation and document these two-dimensional pair of components and this notation is provided to us by Euler with his famous equation:

$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t) \tag{9.8}$$

Where j notes the component at 90° out of phase with the other component, hence the sine and cosine pair. Hence we can replace all sine/cosine pairs at a particular harmonic n with the complex exponential function $e^{jn\omega t}$, and in this way the Fourier series can be rewritten as

$$f(t) = \sum_1^N C_n e^{jn\omega t} \tag{9.9}$$

where C_n represents the resultant from the pair of respective amplitudes for the sine and cosine components. ie: $C_n^2 = b_n^2 + a_n^2$, and is known as the “complex fourier series”.

Euler's formula allows us to process the sine/cosine component pair, simultaneously, rather than individually. It is a form of two-dimensional notation where the sine and cosine components for a frequency are instead treated together as a resultant with particular phase, for example, polar notation. This notation is known as “complex” notation.

Computing Fourier Coefficients

In the previous section it was defined that any arbitrary periodic waveform can be constructed by the summation of a number of sine and cosine harmonics. Now we can explore the reverse process, that is, the analysis of an arbitrary wave form to discover the presence and amplitude of its constituent harmonics or the Fourier coefficients of the waveform as they are the coefficients within the Fourier series equation.

We know that the area under a sine wave or a cosine wave over a timeframe of one period will always be equal to 0. Consequently we know that the area under a sine wave or cosine wave for a timeframe of any number of periods will also be equal to 0. To determine the area mathematically we would integrate over the timeframe of interest. This process would give us the average value of the signal during that time. This average value is the DC component of the signal.

The INTEGRATE & DUMP module can be used to integrate over a single period as denoted by the input clock. This is well suited to integrating periodic waveforms such as sinusoids and products thereof.

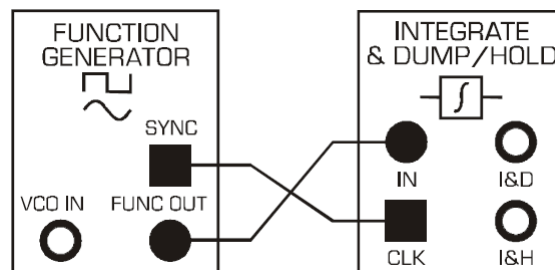


Figure 9-1 Model for Integrating One Period of a Sinusoid

Another way of extracting the DC component of a signal is by using a filter which is tuned to a sufficiently low-frequency to exclude all harmonics except for the DC component.

PRELAB

Constructing Waveforms from Sines and Cosines

The “harmonic summer” simulation in Experiment TAB 8 can be used to view and sum multiple signals which are harmonics of the fundamental signal ie: the signal for $n=1$. Harmonics for which n is even are known as even harmonics, and when N is odd are called odd harmonics. This simulation allows us to view and sum any sinusoid for which $1 \leq n \leq 10$, that is, the fundamental and nine harmonics. The numeric entry boxes allow you to enter the amplitude of each sinusoid.

1. Experiment a little by entering various amplitudes into the numeric entry boxes for the cosine row. Confirm for yourself visually that what is displayed is as you would expect.
2. Set all amplitudes to zero, except for the “1st” component, the fundamental, which you can set to 1.
3. Moving from the second to the ninth sequentially, set each amplitude to 1. As you do this notice how the combined signal is changing. Do this for the cosine row ONLY at this point.
4. Question 1: How would you expect the summation of to look if you could add up many more harmonics?
5. Question 2: What is its peak amplitude and is this as expected?
6. Question 3: Is the fundamental an odd or even function? Is the summation odd or even?
7. Question 4: Write the equation for the summation of the 10 signals? Is it symmetrical about the X axis?
8. Question 5: Vary the amplitudes and notice how the signal changes. You may set the amplitude of certain components to 0 as you see fit. Can you create a wave form which starts at a zero value? Write the equation for your new varied amplitude signal? Does it start at a zero value?
9. Sketch your arbitrary wave form, for which you have just written the equation in Graph 9-1.
10. Set all cosine wave amplitudes to 0 and now set the sine waveform amplitudes to 1, starting from the first harmonic and moving sequentially until the 10th harmonic. Notice how the summation changes as you add harmonics.
11. Question 6: How would you expect the sine summation of to look if you could add up many more harmonics?
12. Question 7: What is its peak amplitude and is this as expected? Is this an odd or even function?
13. Question 8: Vary the amplitudes and notice how the signal changes. You may set the amplitude of certain components to 0 as you see fit. Can you create a wave form which starts at a non-zero value? Write the equation for your new varied-amplitude signal? Does it start at a non-zero value? Is it symmetrical about X axis?
14. Add a DC component to the signal by inputting the value into the DC numeric entry box.
15. Using the “harmonic summer” simulation in the Experiment 8 TAB of the SIGEx SFP, set all amplitudes including that of the DC equal to 0. Set the amplitude of the sine and cosine first harmonic equal to 1. View the resulting summation.

16. Question 9: Write the equation for the summation of these 2 waves? Write the equation for the summation in terms of the sine wave with a nonzero phase shift.
17. Vary the amplitudes from 0 to 1 and notice how the signal changes.
18. Question 10: Describe how the summation changes as you vary the respective amplitudes?
19. Question 11: For a particular pair of amplitudes you have set, write the equation for the summation in terms of sine and cosine as well as its equivalent polar representation?

Sketch a vector or for phasor representation of these two signals and their resulting summation, known as the resultant in Graph 9-2.

PROCEDURE

Setting up the NI ELVIS/SIGEx Board

1. Turn on the PC (if not on already) and wait for it to fully boot up.
2. Turn on the NI ELVIS unit but not the Prototyping Board switch yet. You should observe the USB light turn on (top right corner of ELVIS unit).
3. Turn on the NI ELVIS Prototyping Board switch to power the SIGEx board. Check that all three power LEDs are on. If not call the instructor for assistance.
4. Launch the “SIGEx Rx_x.exe” Main VI.
5. When you’re asked to select a device number, enter the number that corresponds with the NI ELVIS that you’re using.
6. You’re now ready to work with the NI ELVIS/SIGEx bundle.
7. Select the Exp 5 tab on the SIGEx SFP.

Note: To stop the SIGEx VI when you’ve finished the experiment, it’s preferable to use the STOP button on the SIGEx SFP itself rather than the LabVIEW window STOP button at the top of the window. This will allow the program to conduct an orderly shutdown and close the various DAQmx channels it has opened.

Computing Fourier Coefficients

1. Connect the model as per Figure 9-1 above. The clock connection is necessary for this module, unlike for the continuous time INTEGRATORS on the SIGEx board.
2. Settings are as follow:
 - a. SCOPE: Timebase: 4ms; Trigger on Ch0; Connect Ch0 to CLK input; Level=1V
 - b. FUNCTION GENERATOR: Select SINE output, AMPLITUDE = 4vpp, FREQUENCY=1kHz
 - c. Refer to the SIGEx User Manual for instructions on how the I&D and I&H functions work if this is not obvious. NOTE: The I & D period is from positive clock edge to positive clock edge.
3. Question 12: What is the output value at the end of the integration period ? HINT: the I&H function will hold the final value.
4. While viewing the I&D output, to broaden your understanding of the integration process, change the incoming signal from sine to triangle to bipolar squarewave at the FUNCTION GENERATOR VI and confirm that the integrated output is as you would expect over one period. You should be able to confirm that integration is the accumulation of the total signal

charge, with positive signals adding to the total, and negative signals subtracting from the total.

DC Extraction using LPF

Note that in this next part of the experiment the signals we are using are generated from data arrays in LabVIEW and output in a synchronized manner from the ANALOG OUT terminals DAC-0 and DAC-1. These signals are thus synchronized to each other, just as they appear on screen and in the textbook.

1. Using the left hand side of Experiment 8 TAB of the SIGEx SFP, select a sine wave , 1st harmonic. Connect DAC-1 to the input of the TUNEABLE LPF and view both input and output with the scope.
2. Settings are as follow:
 - a. SCOPE: Timebase 4ms; Rising edge trigger on CH0; Trigger level=0V
 - b. TUNEABLE LPF: GAIN=mid position
3. Adjust the frequency control, f_c , of the TLPF from fully clockwise moving counterclockwise until the output signal is a DC value of approximately 0 V.
4. Try this test for several harmonics. Change the “sine harmonic” value to 2,3 or 5 etc to try this.
5. Set the “sine harmonic “ value back to 1. Set “sine phase” to 0. View DAC-1 with CH1 and DAC-0 with CH0.
6. Settings as follows:
 - a. SCOPE: Timebase 4ms; Rising edge trigger on CH0; Trigger level=0V
 - b. DAC-1 will be a sine wave and DAC-0 a cosine wave, relative to each other.
7. Connect as shown in the Figure 9-2 below. View the output of the MULTIPLIER as well as the output of the TUNEABLE LOW PASS FILTER.

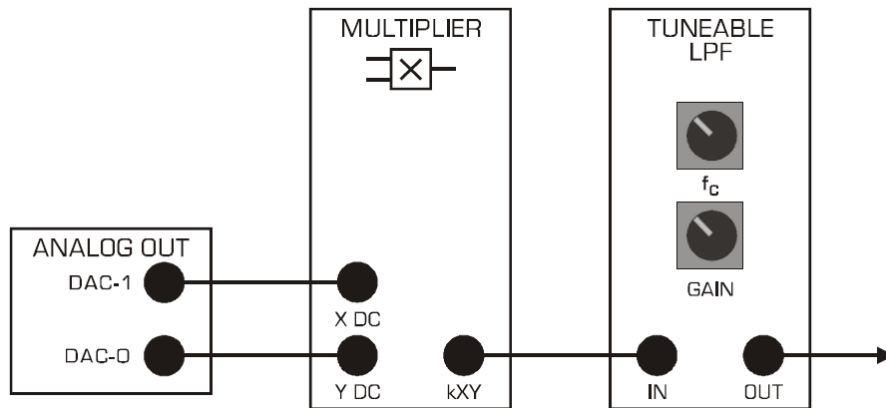


Figure 9-2 Comparing Sinusoids

8. You will now multiply a sine wave by a cosine wave and determine its average value. Repeat this with the second and third harmonic of the sinusoid.
9. Question 13: What is the average value of these three products ?

10. Set the “sine harmonic “ value back to 1. Set “sine phase” to 90. View DAC-1 with CH1 and DAC-0 with CH0.
11. Settings as follows:
 - a. SCOPE: Timebase 20ms; Rising edge trigger on CH0; Trigger level=1V
 - b. DAC-1 will now be a cosine wave and so will be DAC-0.
 - c. NB: This is relative to the start of the signal data array. See the ANALOG OUT VIEWER window for confirmation.
 - d. Connect as shown in the Figure 9-2 above.
12. View the output of the MULTIPLIER as well as the output of the TUNEABLE LOW PASS FILTER with the scope.
13. You will now multiply a cosine wave by itself and determine its average value.
14. Question 14: What is the average value of the product of a cosine by itself?
15. Question 15: Write the complete formula for the product of a cosine, \cos^2 , by itself? What do the terms represent?

At this point we can see that the only product to yield a non-zero average value is that of a sine wave by the same sine wave. The same is true for cosine. So if we multiply an arbitrary waveform by a particular “probing” sine or cosine harmonic and extract the average value of this product there will only be non-zero when that arbitrary wave form contains that particular “probing” harmonic as one of its constituent components. This is a very powerful insight and allows us to have a simple tool with which to analyse arbitrary wave forms.

16. Construct the following arbitrary wave form using the HARMONIC SUMMER on the right hand side of the SIGEx SFP, Experiment 8 TAB.
17. Settings are as follows :
 - a. Cosine amplitudes: 1, 0, 0.5,0,0,1,0,0,0,0
 - b. Sine amplitudes: 0,0.3,1,0,0,0,2,0,0,0
 - c. DC level: 0.5
 - d. Switch “to DAC-0” ON: This will output the summation to DAC-0. See the ANALOG VIEWER window.
 - e. Set “sine harmonic” =1; Set “sine phase = 0. This signal is output to DAC-1. SCOPE: Timebase 4ms; Rising edge trigger on CH0; Trigger level=0V
18. You will need to now ensure that the overall gain of the MULTIPLIER and TLPF is unity.
 - a. Connect both MULTIPLIER inputs to the same 1st harmonic sinewave and view the output. While viewing the TLPF output, turn the fc control clockwise to pass the entire signal. Adjust the TLPF gain precisely for a 4Vpp output, so as to have unity gain throughout the “analyser” path.
19. Return to your previous setup but do not touch the TLPF GAIN setting for the rest of the experiment. At this point we are multiplying the arbitrary wave form which investigated by a first harmonic sine wave.
20. View the product at the output of the multiplier on CH0 and the TLPF output. Adjust the TLPF frequency control to isolate the DC component. Note the value of the DC level as the amplitude of the first harmonic sinusoid in Table 9-1 below. Increase harmonics one by one,

adjust the TLPF frequency setting if necessary and note the amplitude for the remaining 10 sine harmonics in table.

21. Set “sine harmonic” back to 1. Change “sine phase” to 90. This will convert the sine wave into a cosine wave, as $\sin(\omega t + 90) = \cos(\omega t)$. Repeat the above steps to all 10 cosines and enter your measurements of the TLPF DC output level into the table.
22. Connect the input of the TLPF directly to the arbitrary wave form at DAC-0 to measure the DC value of that waveform alone, and enter into the table at “DC (V) = ”.
23. While you have the experiment setup, vary the harmonic summer settings and see the resulting output from the TLPF. Experiment a little with values while you have the system setup and ready.

DATA/OBSERVATIONS

Graph 9-1 Sine Wave Summation

INSTRUCTOR'S INITIALS:

DATE:

Graph 9-2 Components and Resultant

INSTRUCTOR'S INITIALS:

DATE:

TABLE 9-1 Measured Coefficients

Harmonic number	sine (V)	cosine (V)
1st		
2nd		
3rd		
4th		
5th		
6th		
7th		
8th		
9th		
10th		
DC (V) =		

INSTRUCTOR'S INITIALS:

DATE:

POSTLAB

Post-Lab questions must be answered in each experiment's laboratory report.

1. How do your readings compare with expectations? Explain any discrepancies.