Show all your work (derivations and calculations). Clearly indicate the question and part number for all your answers. Label all your plots.

Formulas:
- Integration by parts: \( \int udv = uv - \int vdu \).
- Exponential Fourier Series: \( x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} \).
- Fourier series coefficients: \( a_k = \frac{1}{T} \int_T x(t) e^{-jk\omega_0 t} dt \).
- Differentiation property: When \( a_k \) are the exponential Fourier series coefficients for \( x(t) \), \( jk\omega_0 a_k \) are the Fourier coefficients for \( \frac{dx(t)}{dt} \).
- Fourier transform: \( X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt \).
- Inverse Fourier transform: \( x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega) e^{j\omega t} d\omega \).
- Parseval Equality: \( \int_{-\infty}^{+\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |X(j\omega)|^2 d\omega \).

1. A periodic signal \( x(t) \) is given graphically below:
   (a) What is the value of \( \omega_0 \)?
   (b) Compute the exponential Fourier series coefficients for \( x(t) \).
   (c) Compute exponential the Fourier series coefficients for \( x_1(t) = x(t - 0.5) \).
   (d) Compute exponential the Fourier series coefficients for \( x_2(t) = \frac{dx(t)}{dt} \).
2. You are given Fourier series coefficients that represent a periodic input signal $x(t)$ whose fundamental frequency is $\omega_0 = \pi$. Given the nonzero Fourier series coefficients for $x(t)$ are $a_0 = -1$, $a_1 = a_{-1} = -\frac{1}{2}$ and $a_2 = a_{-2} = 1$, compute the time domain signal, $x(t)$, given by summing the Fourier series terms.

3. The spectrum of a signal $g(t)$ is given by

$$G(j\omega) = \begin{cases} 1, & -1 \leq \omega \leq 1 \\ 0, & \text{elsewhere.} \end{cases}$$

(a) What is $g(t)$

(b) Sketch the spectrum of $y(t) = g^2(t)$.

(c) Sketch the spectrum of $z(t) = g(t) \cos(20t)$.

(d) Determine the total energy of the signal $z(t) = g(t) \cos(20t)$. 