# University of North Carolina at Charlotte Department of Electrical and Computer Engineering

Laboratory Experiment Report #1

**Experiment #1 Thevenin Equivalence** 

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## **Objectives**

The objective of this experiment was to compare voltage, current, and power measurements of a complex network to the values measured from the Thevenin equivalent version of the same circuit.

## **Equipment List**

- HP E3612A DC Power Supply
- Agilent 34401A 6 ½ Digit Multimeter
- CTI SB-147 Breadboard

# **Relevant Theory/Background Information**

The Thevenin equivalent circuit of a network allows a complex circuit to be transformed to a simple circuit of an ideal voltage source in series with a source resistance. The ideal voltage source has a value equal to the voltage appearing at the output of the complex circuit when the load resistance is removed (i.e. replaced with an open circuit). The voltage source is therefore called VOC, meaning open-circuit voltage. The series source resistance of the Thevenin equivalent circuit is the resistance seen looking into the terminals of the complex circuit at the load. This resistance is labeled Rthev or Rth which stands for Thevenin equivalent resistance. Many methods exist for find the Thevenin equivalent circuit. One method is as follows:

- 1. Calculate the open circuit voltage, VOC. This step involves locating the load resistance and removing it from the circuit, then calculating the voltage that appears at the open circuit left by its absence.
- 2. Calculate the short circuit current, ISC. This step involves replacing the load resistance with a short circuit and determining the current that flows in the short.
- 3. Calculate the Thevenin resistance, Rth. This step is simply using Ohm's law using the voltage and current from the first two steps. The Thevenin resistance is Rth = VOC / ISC.
- 4. Draw the Thevenin equivalent circuit.

In this experiment the voltage and current of the load resistance is measured in a bridge circuit shown in Figure 1-1a. The circuit is then replaced with the Thevenin equivalent circuit shown in Figure 1-1b and the voltage and current of the load resistance is recorded again. The values of these measurements are expected to be within a small percent error.

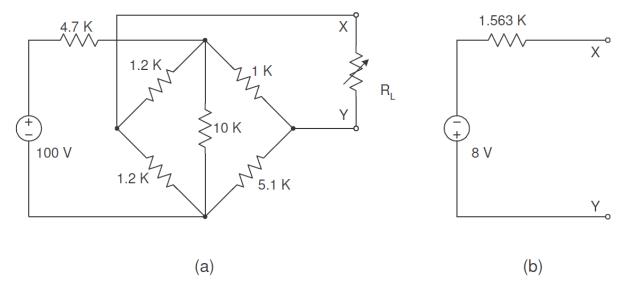


Figure 1-1 Circuit schematics (a) Bridge network (b) Thevenin equivalent

## **Experimental Data/Analysis**

The results from measuring the voltage across the load resistance and the current through the load resistance were tabulated in Table 1-1 along with the calculated values from the prelab exercise. Percent error calculations were also included to analyze this data. The percent error reviled a close correlation in the measured and calculated values, assuring that the circuit was constructed correctly. The voltage and current measurements from node X to Y (i.e. VXY) resulted in negative values but the absolute values were recorded in the table with the understanding that the same result was produced. The difference in the results was due to the tolerance of the resistors and human error in reading the multimeter. Heating of the resistors could have also caused some error but unlikely due to the low level of power used.

Table 1-1
Measured and calculated voltage and current values for the bridge network

	Voltage			Current		
Resistance	Bridge	Calc.	Error	Bridge	Calc.	Error
(Ohms)	(volts)	(volts)	(%)	(mA)	(mA)	(%)
100 k	8.00	7.88	1.56	0.00	0.08	-100
40 k	7.60	7.70	-1.29	0.185	0.19	-3.89
10 k	6.75	6.92	-2.44	0.80	0.69	15.6
5 k	6.00	6.10	-1.56	1.35	1.22	10.74
1.563 k	4.00	3.93	2.08	2.70	2.61	3.36
1 k	3.10	3.12	-0.70	3.20	3.12	2.50
500	1.90	1.94	-2.03	4.10	3.88	5.70
100	0.45	0.48	-6.48	5.00	4.81	3.90
50	0.26	0.25	4.81	5.15	4.96	3.00
0	0.00	0.00	0.00	5.18	5.12	1.17

(Note: All the voltage and current values are negative.)

A large range of resistance values was tested in order to verify the V-I characteristic of the circuit. The Thevenin resistance of the circuit was also used as one of the load resistances to later show the maximum power transfer relation.

The results of the Thevenin equivalent circuit was recorded in Table 2 along with values from the bridge circuit and percent errors. Again the results from the two circuits were determined to have a close correlation proving that a network and its Thevenin equivalent have the same I-V characteristics. Basically, Thevenin's theorem has been verified for the example bridge network using laboratory measurement techniques.

Table 1-2
Measured voltage and current values for the bridge network and equivalent Thevenin's Network

	Voltage		Current			
Resistance	Bridge	Thevenin	Error	Bridge	Thevenin	Error
(Ohms)	(volts)	(volts)	(%)	(mA)	(mA)	(%)
100 k	8.00	8.00	0.00	0.00	0.00	0.00
40 k	7.60	7.70	1.32	0.185	0.175	-5.41
10 k	6.75	6.80	0.74	0.80	0.67	-16.3
5 k	6.00	6.15	2.50	1.35	1.25	-7.41
1.563 k	4.00	3.90	-2.50	2.70	2.70	0.00
1 k	3.10	3.05	-1.61	3.20	3.20	0.00
500	1.90	2.00	5.26	4.10	3.95	-3.66
300	-	1.30	-	-	4.40	-
100	0.45	-	-	5.00	-	-
50	0.26	0.26	0.00	5.15	5.10	-0.97
0	0.00	0.00	0.00	5.18	5.14	-0.77

(Note: All the voltage and current values are negative.)

A mistake was made in the experiment seen by the missing data in Table 1-2. A load resistance of  $100~\Omega$  was used in the bridge circuit of Figure 1-1a, and a resistance value of  $300~\Omega$  was used in the Thevenin's equivalent circuit of Figure 1-1b. This mistake in bookkeeping had little impact on the conclusions of this experiment.

The power produced at the load resistance was calculated and recorded in Table 1-3. The maximum power at the load resistance can be seen with the load resistance of 1.563 k $\Omega$ . The result was the same for both the Bridge and Thevenin circuit. This value was equal to the Thevenin resistance proving the observation of maximum power transfer. This data also again verifies that a circuit and its Thevenin equivalent will have the same I-V characteristics.

Table 1-3
Comparison of the measured output power of the bridge network and equivalent Thevenin's Network to the calculated value.

	Power					
Resistance	Bridge	Thevenin	Calc.			
(Ohms)	(mW)	(mW)	(mW)			
40 k	1.41	1.35	1.48			
10 k	5.40	4.56	4.79			
5 k	8.10	7.69	7.43			
1.563 k	10.8	10.5	10.2			
1 k	9.92	9.76	9.75			
500	7.79	7.90	7.62			
50	1.29	1.33	1.23			

The recorded data from the experiment was further analyzed by constructing Figures 1-2 and 1-3 from the tabulated data. A voltage versus current plot was constructed in Figure 1-2 showing data points from both circuits as well as the calculated values. A linear load line was drawn on the plot approximating in the data collected from the experiment. Two points from the load line were used to calculate the source resistance of the network to be 1.548 k $\Omega$  which has a 0.96% error when compared to the Thevenin equivalent resistance  $R_{th}$ .

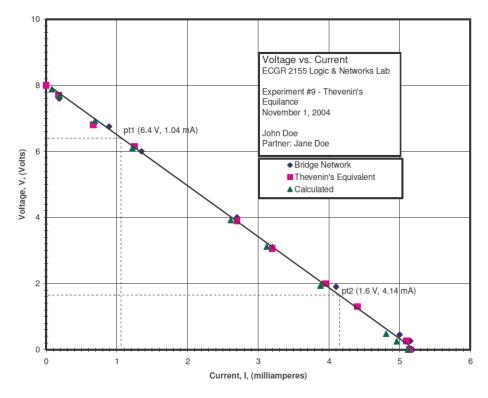


Figure 1-2 Voltage vs. Current

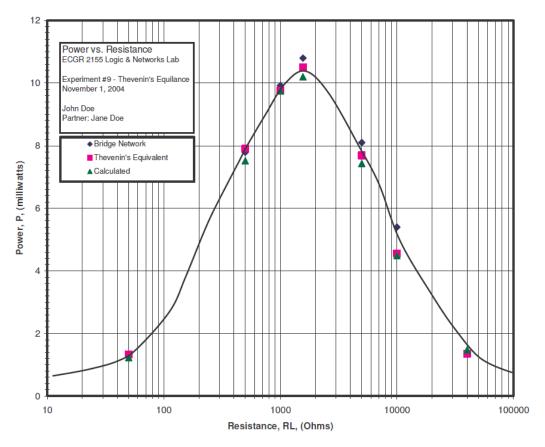


Figure 1-3 Power vs. Resistance

The power delivered to the load resistance was plotted in Figure 1-3. This graph was made with a semi-log scale for the x-axis to show the characteristic bell-shape pattern of the maximum power transfer theorem. The curve was then added by sketching a line between the data point. As expected the maximum power is delivered to the load when it is equal in resistance to the resistance of the looking into the circuit, in this case, the Thevenin equivalent resistance.

## Sample Calculations

1. Sample calculation of the voltage across the load resistance  $R_L$  using the Thevenin's equivalent network from the prelab exercise.

$$V(R_L) = V_{th} \left[ \frac{R_L}{R_L + R_{th}} \right]$$

$$V(1563) = (-8) * \left[ \frac{1563}{1563 + 1500} \right] = -3.92 \text{ volts}$$

2. Sample calculation of current through the load resistor  $R_L$  using the Thevenin's equivalent network from the prelab report.

$$I(R_L) = \frac{V_{th}}{R_L + R_{th}}$$

$$I(R_L) = \frac{-8}{1563 + 1500} = -2.61$$
 milliamperes

3. Sample calculation of power using bridge measurement data for a load resistance  $R_L$  of 1.563 k $\Omega$ .

$$P = VI = (4.00)(2.70) = 10.8$$
 milliwatts

4. Sample calculation of error between measured voltage of the bridge network and calculated voltage for a load resistance  $R_L$  of 1.563 k $\Omega$ .

Percent Error = 
$$(100) (V_{bridge} - V_{calc}) / (V_{calc})$$

Percent Error = 
$$(100) (4.00 - 3.92) / (3.92) = 2.08\%$$

5. Sample calculation of the source resistance of the bridge network and Thevenin equivalent circuit using data form a load line.

$$R_{\text{source}} = \text{slope of line} = \left| \frac{\text{rise}}{\text{run}} \right| = \frac{|y_1 - y_2|}{|x_1 - x_2|}$$
 where x and y values are points on the plot.

$$R_{\text{source}} = \frac{|6.4 \text{ V} - 1.6 \text{ V}|}{|1.04 \text{ mA} - 4.14 \text{ mA}|} = \frac{4.8 \text{ V}}{3.1 \text{ mA}} = 1548 \text{ Ohms}$$

#### **Conclusions**

A linear resistive network can be replaced by a Thevenin's equivalent network made of a voltage source in series with a resistance. The value of the Thevenin's equivalent voltage would be the open-circuit voltage measured at the output terminals of the original network. The Thevenin's equivalent resistance would be the resistance seen looking back into the output terminals of the original network with all voltage sources shorted and all current sources opened. Maximum power transfer to a resistive load across the output terminals of a linear resistive network occurs when the load resistance equals the Thevenin's equivalent resistance of the network.

#### **List of Attachments**

Original data sheets (1 and 2)

#### References

1. Giorgio Rizzoni. *Principles and Applications of Electrical Engineering*. 2<sup>nd</sup> Edition, Irwing, Chicago, IL 1996.