Problem 1 – Consider the ordinary differential equation \( y' = x^2 - y^2 + 1 \)

A – Show that, with the change of variable \( u = \frac{1}{y-x} \)
the equation becomes \( u' = 2xu + 1 \)

B – Solve the new equation for \( u \), and find the solution for which \( u(0) = a \)

Problem 2 – The half life of carbon-14 is 5730 years. A sample of charcoal found at Stonehenge has about 63% as much carbon-14 as a sample of present day charcoal of equal mass. What is the approximate age of the sample from Stonehenge?

Problem 3 – A mass-spring-dashpot system is modeled by the equation
\( x'' + 2px' + \omega_0^2x = 0 \) where \( p \) is the damping constant and \( \omega_0 \) is the undamped angular frequency. Assume that the system is underdamped \( p^2 - \omega_0^2 < 0 \)

A – Derive an expression (in terms of \( p \) and \( \omega_0 \)) for the “pseudo angular frequency” \( \omega_1 \) that appears in the solution \( x = Ae^{-pt} \cos(\omega_1 t - \phi) \)

B – What is the value of \( p/\omega_0 \) if \( \omega_1 = \omega_0/3 \)?