Problem 1  

**Engineering Mathematics**  
P2: The Z-transform

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\[ X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \]

\[ X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \]

\[ u[n] \text{ is the unit step, } \delta[n] \text{ is the unit sample} \]

\[ \omega \text{ denotes frequency in rad/sample} \]

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**Circle the Best Answer**  
**Show All Work Even For Multiple Choice**

1. The right-sided sequence \( h(n) \) with z-transform \( H(z) = \frac{2z-1}{z^2+3z+2} \) is BIBO stable.
   
   a) True  
   b) False

2. If a filter has \( H(z) = \frac{z^2+2z-2}{4z^2-1} \); \(|z|>1/2\), then the dc response of the filter at \( \omega=0 \) is
   
   a) 0  
   b) 1/5  
   c) 1/3  
   d) 1/2  
   e) none above

3. The ROC  \( 0.7 < |z| < 1.1 \) could be associated with a BIBO stable 2-sided sequence.
   
   a) True  
   b) False

4. If \( H(z) = \frac{z^2-z/3-1/2}{z^2/4-1/16} \), then the poles of \( H(z) \) are at \( z= \)
   
   a) \( j/4, -j/4 \)  
   b) \( j/2, -j/2 \)  
   c) \( 1/2, -1/2 \)  
   d) none above

5. One of the zeroes of the z-transform \( H(z) = 1 + z^{-3} \) is at \( z= \)
   
   a) \( e^{j\pi/2} \)  
   b) \( e^{j3\pi/8} \)  
   c) \( e^{-j\pi/3} \)  
   d) \( e^{j5\pi/4} \)  
   e) none above
6. The z-transform of \( h[n] = \delta[n-1] - \frac{1}{4} \delta[n-3] \) is \( H(z) = \)

a) \( 1 + 4z^{-3} \); \(|z| > 0.25\)  
b) \( \frac{z^2 - 4}{z^3} \); \(|z| > 0\)  
c) \( \frac{4z^2 - 1}{4z^3} \); \(|z| > 0\)  
d) none above

7. The z-transform of \( h[n] = (1/2)^{n-1} u[n] \) is \( H(z) = \)

a) \( \frac{2z}{z - 1/2} \); \(|z| > 1/2\)  
b) \( \frac{2}{2z - 1} \); \(|z| > 1/2\)  
c) \( \frac{1}{2z - 1} \); \(|z| > 1/2\)  
d) none above

8. The system with z-transform \( H(z) = \frac{(z+1)}{z^2 + 0.0001} \); \(|z| > 0.01\) would be best described as

a) lowpass  
b) highpass  
c) bandpass

9. If a filter has \( H(z) = \frac{z^3 - 2z^2 - 1.9}{z^3 + 0.1} \) and ROC \(|z| > 0.1^{1/3}\), then the first 3 points of \( h[n] = \)

a) \( \{1, -2, 1.9\} \)  
b) \( \{1, 0, -2.0\} \)  
c) \( \{0, 1, -2.0\} \)  
d) \( \{1, -2, 0\} \)  
ed) none above

10. The z-transform of \( u[n] - u[n-3] \) is

a) \( 1 - 2z^{-1} - 3z^{-2} \); \(|z| > 1/5\)  
b) \( z^{-1} + z^{-2} \); \(|z| > 0\)  
c) \( 1 - z^{-1} - z^{-2} \); \(|z| > 0\)  
d) none above

11. If a system has \( H(z) = \frac{5z-1}{16z^2 + 1/16} \) and ROC \(|z| > 1/16\) then, then \( h[1] = \)

a) -16  
b) 5/16  
c) 5  
d) 16  
e) none above
12. If a filter has impulse response \( h[n] = u[n] - u[n-4] \), the filter response at frequency \( \omega = \pi \) is \( H(\omega) \big|_{\omega=\pi} = \)
   a) -1  
   b) \( e^{-j\pi} \)  
   c) 1  
   d) j  
   e) none above

13. If a filter has \( H(z) = \frac{z^2-z/3-1/2}{z^2+z/6-1/6} \); \( |z|>1/2 \), then response of the filter at \( \omega = \pi \) is
   a) 0  
   b) -1/6  
   c) 3/4  
   d) 5/4  
   e) none above

14. If \( H(z) = \frac{5}{z+1/2} + \frac{1}{z-1/2} \); \( |z|>1 \), then the zero of \( H(z) \) is at \( z = \)
   a) 1/3  
   b) 1/4  
   c) 1/2  
   d) 1  
   e) none above

15. If \( H(z) = \frac{8z}{2z+1} \); \( |z|>1/2 \), then \( h[n] = \)
   a) \( (1/8)^n u[n] \)  
   b) \( 4(-1/2)^n u[n] \)  
   c) \( 2(-1/2)^n u[n] \)  
   d) \( 8(1/2)^n u[n] \)  
   e) none above

16. If a filter has impulse response \( h[n] = (1/5)^{n-1} u[n-1] \), the dc response of the filter is \( H(\omega) \big|_{\omega=0} = \)
   a) 5/4  
   b) 5/6  
   c) 4/5  
   d) 0.2e^{-j2\omega}  
   e) none above

17. A filter with \( H(z) = \frac{z^2+4z+1}{z^2+1/4} \); with ROC \( |z|>1/2 \) is
   a) unstable  
   b) left-sided  
   c) two-sided  
   d) causal  
   e) none above

18. If \( Y(z) = 1+z^{-1}; |z|>0 \), and \( X(z) = 1+z^{-1}; |z|>0 \), then the convolution \( x[n] \ast y[n] = \)
   a) \( \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] \)  
   b) \( \delta[n] + 2\delta[n-1] + \delta[n-2] \)  
   c) \( \delta[n-1] + \delta[n-2] \)  
   d) none above
For the following questions:

\[ H(z) = \frac{z^2 + 1/4}{z^2 + z/2 + 1/16} \]

19. Sketch the poles and zeroes in the figure above.

20. Assuming a causal system, sketch the region convergence in the figure above.

**Show your work for the 2 above problems in the space below.**