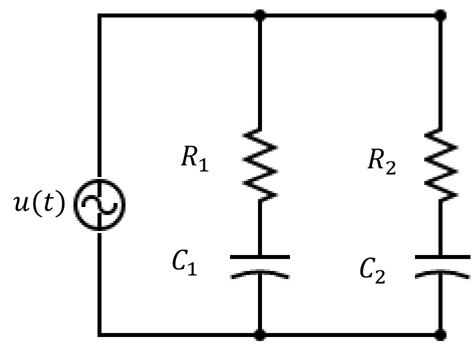
Department of Electrical and Computer Engineering

Fall 2023 COMPREHENSIVE/BREADTH EXAM

Questions 2/3/4 **TTG Area: Comm., Ctrls., and Sig. Proc.** **ECGR 4111: Control Systems**

Solve either Problem A or Problem B.

A. Consider the following RC circuit:



Let $x_1(t)$ denote the voltage across capacitor C_1 and $x_2(t)$ denote the voltage across C_2 at time t. The input to this circuit is a voltage source, where $u(t)$ denotes the voltage input at time t. Suppose you have two sensors to measure the voltage drops across the resistors R_1 and R_2, respectively.

i. Define $X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$ and $Y(t)$ to be the sensor outputs. Derive a state-space model of the given circuit. That is, express the system as

$$\frac{dX(t)}{dt} = AX(t) + Bu(t),$$
$$Y(t) = CX(t) + Du(t),$$

where clearly mention what are A, B, C and D in your state-space model.

ii. Determine if the system is stable for all possible values of R_1, R_2, C_1, C_2. If not, find the least restrictive conditions on R_1, R_2, C_1, C_2 to ensure that the system in stable.

iii. Determine if the system is controllable for all possible values of R_1, R_2, C_1, C_2. If not, find the least restrictive conditions on R_1, R_2, C_1, C_2 to ensure that the system in controllable.

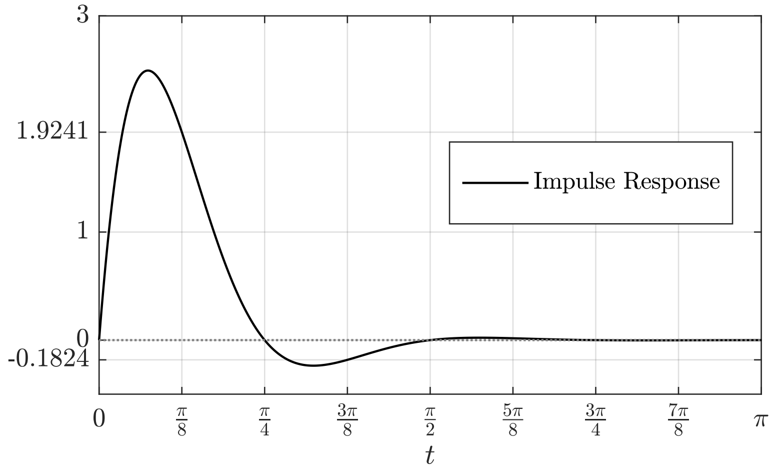
iv. Determine if the system is observable for all possible values of R_1, R_2, C_1, C_2. If not, find the least restrictive conditions on R_1, R_2, C_1, C_2 to ensure the system in observable.

Useful formulas and Equations:

(1) If $v(t)$ is the voltage drop across a capacitor and $i(t)$ is the current through it, then $C\frac{dv(t)}{dt} = i(t)$, where C is the capacitance.

(2) KVL in the first loop is: $u(t) = I_1(t)R_1 + x_1(t)$, where $I_1(t)$ is the current through R_1 at time t. (Similar equation for the second loop involving R_2, C_2.)

B. We have a second order system $\frac{Y(s)}{U(s)} = \frac{25}{s^2+10\zeta s+25}$ where the damping coefficient ζ is not given. However, the impulse response of the system is given in the following figure:



Find the Bandwidth (BW) and resonant peak (M_r) for this system (simplify as much as possible).

Useful formulas: For a second order prototype system $\frac{Y(s)}{U(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$,

(a) The impulse response is: $\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\left(\omega_n\sqrt{1-\zeta^2}\,t\right)$.

(b) The Bandwidth formula is $BW = \omega_n\left[1-2\zeta^2+\sqrt{(1-2\zeta^2)^2+1}\,\right]^{\frac{1}{2}}$.

(c) Resonant peak formula is $M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$ for $\zeta < 0.707$, otherwise $M_r = 1$.