Show all your work (derivations and calculations). Clearly indicate the question and part number for all your answers. Label all your plots.

Formulas:
Integration by parts: \( \int u dv = uv - \int v du \).
Exponential Fourier Series: \( x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{j\omega_0 t} \).
Fourier series coefficients: \( a_k = \frac{1}{T} \int_{-T}^{T} x(t) e^{-j\omega_0 t} dt \).
Differentiation property: When \( a_k \) are the complex exponential Fourier series coefficients for \( x(t) \), \( jk\omega_0 a_k \) are the complex exponential Fourier coefficients for \( \frac{dx(t)}{dt} \).
Fourier transform: \( X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt \).
Inverse Fourier transform: \( x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega) e^{j\omega t} d\omega \).
Parseval Equality: \( \int_{-\infty}^{+\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |X(j\omega)|^2 d\omega \).
Multiplication property: \( s(t)p(t) \leftrightarrow \frac{1}{2\pi} S(j\omega) * P(j\omega) \), where * denotes the convolution operation.

1. A periodic signal \( x(t) \) of period \( T = 4 \) is defined over a period by

\[
x(t) = \begin{cases} 
0 & -2 < t < -1 \\
1 & -1 < t < 1 \\
0 & 1 < t < 2 
\end{cases}
\]

(a) What is the value of fundamental frequency \( \omega_0 \) in rad/s?
(b) Compute the complex exponential Fourier series coefficients for \( x_1(t) = \frac{dx(t)}{dt} \)
(c) Compute the complex exponential Fourier series coefficients for \( x(t) \) using your results from (b)
2. A continuous-time periodic signal $x(t)$ is real valued and has a fundamental period $T = 4$. The nonzero complex exponential Fourier series coefficients for $x(t)$ are specified as

$$a_1 = a_{-1} = 2, \quad a_3 = a_{-3}^* = 4j.$$

Express $x(t)$ in the form

$$x(t) = \sum_{k=0}^{\infty} A_k \cos(\omega_k t + \phi_k).$$
3. You are told that the spectrum of the signal \( g(t) = \frac{\sin(At)}{\pi t} \) is given by

\[
G(j\omega) = \begin{cases} 
1, & -A \leq \omega \leq A \\
0, & \text{elsewhere.}
\end{cases}
\]

Now let

\[
h_1(t) = \left( \frac{\sin(\frac{\pi t}{2})}{\pi t} \right) \left( \frac{\sin(\pi t)}{\pi t} \right).
\]

(a) Compute the total energy \( E_\infty \) of \( g(t) \), where

\[
E_\infty = \int_{-\infty}^{+\infty} |g(t)|^2 \, dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |G(j\omega)|^2 \, d\omega.
\]

(b) Determine the frequency response \( H_1(j\omega) \) for \( h_1(t) \); using the Fourier transform multiplication property. (The formula is given to you on the first page).

(c) Sketch the magnitude \( |H_1(j\omega)| \).

(d) Let \( h_2(t) = h_1(t) \cos(\pi t) \), determine the frequency response \( H_2(j\omega) \) for \( h_2(t) \). (Use the multiplication property given)

(e) Sketch the magnitude \( |H_2(j\omega)| \).