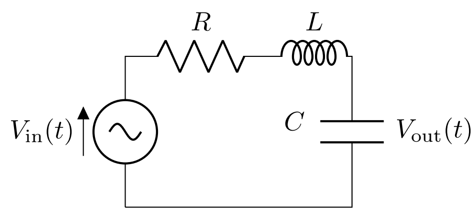
**Control Number:**________________________

Electrical and Computer Engineering
**COMPREHENSIVE/BREADTH EXAM**
TTG Area: Comm., Ctrls., and Sig. Proc. ECGR 4111: Control Systems

**Solve either Problem 1 or Problem 2.**

Let $I(t)$ denote the current in the following series RLC circuit:




The initial voltage across the capacitor and the initial current in the inductor are 0. The voltage equation for the circuit is

$$L\frac{\mathrm{d}I(t)}{\mathrm{d}t} + RI(t) + \frac{1}{C}\int_0^t I(\tau)\mathrm{d}\tau = V_{\text{in}}(t), \tag{1}$$

where $V_{\text{in}}(t)$ is the input to the circuit and $V_{\text{out}}(t)$ (i.e., the voltage across the capacitor) is the output.

Some Useful Formulas

A linear system $\dot{X} = AX + BU, \quad Y = CX + DU$ is *controllable* if

$$\text{rank}([B, AB, \ldots, A^{n-1}B]) = n,$$

where $n$ is the dimension of the state $X$.

The linear system is *observable* if

$$\text{rank}\left(\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}\right) = n.$$

For a second order transfer function of the form $G(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n+\omega_n^2}$, the bandwidth formula is

$$\text{BW} = \omega_n\left(1 - 2\zeta^2 + \sqrt{(1-2\zeta^2)^2+1}\right)^{\frac{1}{2}}$$

The Resonant peak ($M_r$) formula is

$$M_r = \begin{cases} \frac{1}{2\zeta\sqrt{1-\zeta^2}}, & \zeta < \frac{1}{\sqrt{2}} \\ 1, & \text{otherwise.} \end{cases}$$

1. (100 points) State-space analysis.

   (a) (30 points) For the given circuit, derive a state-space formulation of the following form:

   $$\dot{X}(t) = AX(t) + BV_{\text{in}}(t)$$
   $$V_{\text{out}}(t) = CX(t) + DV_{\text{in}}(t),$$

   where $X(t) \in \mathbb{R}^2$. Clearly define your choice of $X(t), A, B, C$ and $D$.

   (b) (20 points) Check the stability of the system and determine whether the system is *marginally stable/ asymptotically stable/ unstable.*

   (c) (5 points) Is it possible to pick $R, L, C$ to make the system unstable? If yes, provide an example. If no, prove your claim.

   (d) (5 points) Is it possible to pick $R, L, C$ to make the system marginally stable? If yes, provide an example. If no, prove your claim.

   (e) (10 points) For your choice of $A, B, C, D$, determine whether the system is controllable?

   (f) (10 points) For your choice of $A, B, C, D$, determine whether the system is observable?

   (g) (20 points) Design a state feedback controller $V_{\text{in}}(t) = -KX$ to place the eigenvalues of the closed-loop system at $-1$ and $-2$ respectively. Express $K$ in terms of $R, L$, and $C$.

2. (100 points) Frequency Domain analysis.

   (a) (15 points) Compute the resonant peak $M_r$ for the given RLC circuit.

   (b) (15 points) Keeping $L$ and $C$ fixed, if we vary $R$, how does $M_r$ change with $R$ (e.g., does $M_r$ increases when $R$ is increased etc.)?

   (c) (15 points) Find the Bandwidth (BW) of the RLC circuit.

   (d) (15 points) Suppose you cannot change $R$, but you may pick $L$ and $C$. How would you choose $L$ and $C$ to maximize the bandwidth of the system?

   (e) (20 points) Let the input be $V_{\text{in}}(t) = \sin(t)\cos^2(t)$. What would be the steady-state output $V_{\text{out}}(t)$ for this input?

   (f) (20 points) Design a Proportional-derivative (PD) feedback controller to place the poles of the closed-loop system at $-1$ and $-2$. Express the gains $K_p$ and $K_d$ in terms of $R, L$ and $C$.