Electrical and Computer Engineering
Comprehensive/Breadth Exam

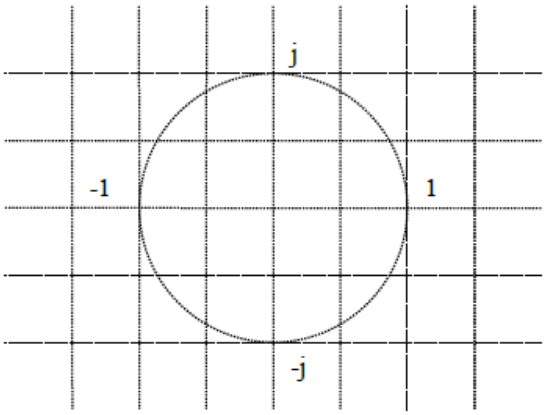
Problem 1 | Engineering Mathematics | P2: The z-Transform

X(z): z-Transform of x[n]	$X[z]=\sum_{-\infty}^{\infty} x[n]z^{-n}$
X(ω): DTFT of x[n]	$X(\omega) = \sum_{-\infty}^{\infty} x[n]e^{-j\omega n}$
u[n]: unit step; δ[n]: unit sample	ω: frequency in rad/sample

1. (i) Compute the z-transform for the following sequence: $\left(-\frac{1}{3}\right)^{-n} u[n] - \left(\frac{1}{2}\right)^{n} u[-n-1]$.

(ii) Sketch the poles and zeros in the figure below.

(iii) Sketch the region of convergence in the figure below.



(iv) Indicate if the sequence corresponds to the unit sample response of a stable system.

Circle the best answer. Show all work even for multiple choice.

2. The right-handed sequence h[n] with z-transform $H(z) = \frac{z-1}{z^2-4}$ is BIBO stable.

   (a) True (b) False

3. The ROC $0.1 < |z| < 0.9$ can be associated with a BIBO stable 2-sided sequence.

   (a) True (b) False

4. For $H(z) = \frac{z^2-4z+4}{4z^2-1}$, the poles are at z=

   (a) ½, −½ (b) 2, 2 (c) j/2, j/2 (b) None above

5. The z-transform of $h[n] = \delta[n] - \delta[n-3]$ is H[z]

   (a) $\frac{1}{1-z^{-3}}$; |z|> 1 (b) $1 - z^{-3}$; |z|≠ 0 (c) $\frac{1}{1-z^{-3}}$; $0 > |z| > 1$ (d) None above

6. The z-transform of $h[n] = \left(\frac{1}{3}\right)^{-n} u[n]$ is H[z]

   (a) $\frac{z}{z-1/3}; |z| > 1/3$ (b) $\frac{3}{z-1/3}; |z| > \frac{1}{3}$ (c) $\frac{1}{1-3z^{-1}}; |z > 3$ (d) Nove abov

7. The z-transform of u[n-1]-u[n-3] is

   (a) $1 - 2z^{-1} - 3z^{-2}$; $|z > \frac{1}{5}$ (b) $z^{-1} + z^{-2}; |z| > 0$
   (c) $1 - z^{-1} - z^{-2}; |z| > 0$ (d) none above

8. If a system has H(z)=$\frac{1}{1-2z^{-1}}$, $|z| > 2$, then h[1]=
   (a) 0 (b) 1 (c) 2 (d) -2 (e) none above

9. If $H(z) = \frac{1}{z+\frac{1}{2}} + \frac{1}{z-\frac{1}{2}}; |z| > \frac{1}{2}$, the zero of H(z) is at z=
   (a) 1/3 (b) ¼ (c) ½ (d)1 (e) none above

10. If $H(z) = \frac{4z}{2z+1}; |z| > \frac{1}{2}$, then h[n]=

(a) $\left(\frac{1}{8}\right)^n u[n]$ (b) $4\left(-\frac{1}{2}\right)^n u[n]$

(c) $2\left(-\frac{1}{2}\right)^n u[n]$ (c) $2\left(\frac{1}{2}\right)^n u[n]$ (e) none above

12. A filter has z-transform $H[z] = \frac{z^2+4z+1}{z^2+4}; |z| <2$. The filter is:

(a) unstable (b) left-sided (c) two-sided (d) causal (e) none-above